

# Distributed Fault Detection for Interconnected Large-Scale Systems: a Scalable Plug & Play Approach

Francesca Boem, Ruggero Carli, Marcello Farina, Giancarlo Ferrari-Trecate, Thomas Parisini

**Abstract**—In this paper, we propose a novel distributed fault detection method to monitor the state of a – possibly large-scale – linear system, partitioned into interconnected subsystems. The approach hinges on the definition of a partition-based distributed Luenberger-like estimator, based on the local model of the subsystems and that takes into account the dynamic coupling between the subsystems. The proposed methodology computes – in a distributed way – a bound on the variance of a properly defined residual signal. This bound depends on the uncertainty affecting the state estimates computed by the neighboring subsystems and it allows the computation of local fault detection thresholds, as well as the maximum false-alarms rate. The implementation of the proposed estimation and fault detection method is *scalable*, allowing *Plug & Play* operations and the possibility to disconnect the faulty subsystem after fault detection. Theoretical conditions on the convergence properties of the estimates and of the estimation error bounds are provided. Simulation results on a power network benchmark show the effectiveness of the proposed method.

## I. INTRODUCTION

In recent years, a growing interest concerned research activities dealing with the design of systems which are reliable and robust with respect to uncertainties, changing environment and communication failures. Methodological developments have been promoted by the new technological paradigm of the Internet of Things [8], [12], which hinges on the massive interconnection of communication networks, sensors, and actuators. Requirements of the Internet of Things in terms of system size and flexibility call for distributed control, monitoring, and fault diagnosis approaches that are well adapted to networked and Large-Scale Systems (LSSs) [24]. In this connection, estimating the state in a distributed way is certainly a key issue to be addressed.

The problem dealt with in the paper consists in estimating the state of a LSS, characterized by interconnected subsystems, and taking decisions about the health status of the system,

using a network of local state estimation and fault detection units equipped with sensing, communication and computation capabilities. Differently from many contributions in the literature, where the full state of the system is estimated by all subsystems, (e.g., techniques based on consensus and diffusion strategies [13], [26], [27], [28], [29], [39]), in this paper each local unit only estimates a part of the global state vector. In more specific terms, a *partition-based distributed* estimation technique is designed in which each local estimation and detection unit exchanges information with the neighboring units thus allowing to estimate the state of the associated local subsystem only.

Recently, several different partition-based approaches have been proposed: for example, [30], [11], [40], [23], [6] propose Kalman-filter-based estimation schemes for discrete-time systems affected by stochastic noise, while [7], [19] assume that the system is affected by bounded noise, and the unperturbed case is considered in [20], [21].

Linear discrete-time large-scale interconnected systems affected by stochastic noises are considered in this paper. The partition-based distributed estimation method introduced in [6] in order to compute an estimate of the local state is significantly extended to address the distributed fault detection problem by proposing a different definition of the correction gains. Then, a consistent upper bound is locally computed for the covariance matrix of the estimation error (see [22] for the formal definition of consistency of covariance estimates) and this bound is used to derive a suitable fault detection threshold for a local residual signal, *aiming at guaranteeing a low false-alarm rate*. A notable feature of the proposed method is that an upper bound on the false-alarm rate can be set depending on design parameters. A novel distributed fault detection procedure is thus obtained.

In the past few years, several distributed model-based fault detection schemes have been proposed, addressing rather different problems. For instance, the reader is referred to [38], [33], [25], [31], [32], [14], [34], [35], [36], [37], [15], [2], where coupled or dynamically independent agents are in charge of detecting faults locally and collectively by means of a set of local detection units, and using either absolute or relative measurements. In [38] faults are explicitly modeled as additive inputs to the state equation with unknown amplitude and a distributed, iterative, and fully connected implementation of a moving horizon estimator is used for fault detection and isolation. On the other hand, most of the mentioned methods consider deterministic bounds for noises and uncertainties in order to suitably determine detection thresholds. Instead, with the aim of achieving a less conservative detection performance, a stochastic characterization of the noises and the definition of time-varying bounds is considered (see also the preliminary

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F. Boem is with the Dept. of Electronic and Electrical Engineering at University College London (UCL), UK. (francesca.l.boem@gmail.com)

R. Carli is with the Dip. di Ing. dell'Informazione, Università di Padova, Italy. (carlirug@dei.unipd.it)

M. Farina is with the Dip. di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Italy. (marcello.farina@polimi.it)

G. Ferrari-Trecate is with the Automatic Control Laboratory, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland. (giancarlo.ferraritrecate@epfl.ch)

T. Parisini is with the Dept. of Electrical and Electronic Engineering at Imperial College London, UK, with the KIOS Research and Innovation Centre of Excellence, University of Cyprus, and with the Dept. of Engineering and Architecture at University of Trieste, Italy. (t.parisini@gmail.com)

works [3], [41]). Moreover, instead of using a sensor network to monitor a system characterized by stochastic uncertainties, where each sensor takes noisy measurements of the entire state [41], in this paper only a part of the state is considered by each local estimation and detection unit thus significantly broadening the applicability of the proposed approach.

When dealing with LSSs, the *scalability* and *Plug & Play* (PnP) properties have a key role [24]. In this respect, an important feature of the proposed methodology is the possibility of unplugging a faulty subsystems in order to avoid or reduce the propagation of faults in the interconnected system, and the possible plug-in of the disconnected subsystem (once the issue has been solved), without the need of a global re-design of the estimators but only resorting to local operations. Compared with [3], in this paper the knowledge of the mean and the variance of the coupling uncertainty is not assumed to be known and the computation of a bound for the influence on the uncertainty of the neighboring estimates is presented. Furthermore, the assumption used in [3] that the state is fully measurable is here removed.

To sum up, the main contributions of the paper<sup>1</sup> are:

- The design of a distributed estimation and fault detection scheme able to address the dynamic coupling between subsystems.
- A recursive equation for computing – in a distributed way – an upper bound on the true covariance matrix of the estimation error, thus allowing the design of a distributed fault detection threshold from which a maximum probability of false alarms can be evaluated at each time step. We also guarantee that this time-varying upper bound converges to a stationary value, under suitable conditions that can be checked in a scalable fashion.
- The PnP feature enabled by the limited amount of information exchanged between neighboring subsystems. More specifically, when a new subsystems issues a plug-in request, (i) the possibility of adding it without spoiling convergence of the estimation scheme is automatically checked and (ii) only subsystems that are at most two-hops-away from the new unit need to update local estimators and fault detectors.

The paper is structured as follows. In Section I, we introduce the distributed estimation and fault detection method. In Section III we propose a bound for the estimation error covariance matrix and we provide some convergence conditions. The scalability features are analyzed in Section IV. Finally, extensive simulation results showing the effectiveness of the distributed fault detection methodology are presented in Section V using a multi-area power network use-case.

**Notation.** Given a stochastic variable  $x$ , we represent as  $\mathbb{E}[x]$  its expected value. The symbols  $\geq$  and  $>$  are used to denote positive semi-definite matrices and positive definite matrices, respectively. The spectral radius of a square matrix  $A$  is  $\sigma(A)$ . Finally, a square matrix is Schur stable if  $\sigma(A) < 1$ .

<sup>1</sup>Preliminary results have been presented in the very recent paper [1]. Compared with [1], a more comprehensive theoretical analysis is provided and extensive numerical results are given. For example, the conservativeness of the bound on the error covariance matrix is analyzed.

## II. DISTRIBUTED FAULT DETECTION: PROBLEM FORMULATION AND PROPOSED SOLUTION

### A. Problem setup

The monitored large-scale discrete-time system is composed of (or can be decomposed in)  $M$  interconnected subsystems. Each subsystem  $\Sigma_i$ , with  $i = 1, \dots, M$ , is described by the following equations:

$$\Sigma_i : \begin{cases} x_i(k+1) = A_{ii}x_i(k) + \sum_{j \neq i} A_{ij}x_j(k) + w_i(k), \\ y_i(k) = C_i x_i(k) + v_i(k), \end{cases} \quad (1)$$

where  $x_i(k), w_i(k) \in \mathbb{R}^{n_i}$  and  $y_i(k), v_i(k) \in \mathbb{R}^{p_i}$ . We assume that  $w_i(k)$  and  $v_i(k)$  are zero-mean white noises, for all  $i = 1, \dots, M$ , and  $\mathbb{E}\{w_i(k)w_j^\top(k)\} = Q_i\delta_{ij}$  (with  $Q_i \geq 0$  for all  $i = 1, \dots, M$ ),  $\mathbb{E}\{v_i(k)v_j^\top(k)\} = R_i\delta_{ij}$  (with  $R_i > 0$  for all  $i = 1, \dots, M$ ), and that  $\mathbb{E}\{w_i(k)v_j^\top(h)\} = 0$  for all  $i, j = 1, \dots, M$  and  $h, k \geq 0$ . Moreover, we assume that the initial condition  $x_i(0)$  is generated according to a Gaussian distribution with mean  $\bar{x}_i$  and covariance  $\Sigma_i$ .

For each  $i \in \{1, \dots, M\}$ , we define the set of neighbors of subsystem  $i$  as  $\mathcal{N}_i = \{j \mid A_{ij} \neq 0, j \neq i\}$  while  $\tilde{\mathcal{S}}_i$  is the set of successors of subsystem  $i$  defined as  $\tilde{\mathcal{S}}_i = \{j \mid i \in \mathcal{N}_j\}$ . For later use, we also define the sets  $\mathcal{N}_i = \mathcal{N}_i \cup \{i\}$  and  $\mathcal{S}_i = \tilde{\mathcal{S}}_i \cup \{i\}$ .

Each subsystem is monitored by a local fault diagnoser. We assume that each diagnoser knows the local dynamic model and can communicate with neighboring subsystems some information that will be defined later on. The goal of each diagnoser is to compute in a *distributed* and *scalable* fashion a local residual which is then compared with a properly designed local threshold in order to take decisions about the healthy status of the subsystem. Specifically, by *distributed* we mean that each diagnoser needs only local and neighbors' information, requiring communication only with neighboring subsystems. Furthermore, also the design needs only local computations, allowing a *scalable* evolution of the LSS over time, where some subsystems may be plugged-in and others may be unplugged, without requiring the reconfiguration of the entire LSS, but only of the neighboring subsystems.

To compute the local residual each diagnoser implements a local estimator which is described in next Subsection, while the design of the local threshold is discussed in Subsection II-C.

### B. Computing the local residuals

We assume each diagnoser locally implements a Luenberger observer to estimate the local state vector:

$$\begin{cases} \hat{x}_i(k+1) = \sum_{j \in \mathcal{N}_i} \{A_{ij}\hat{x}_j(k) + L_{ij}[y_j(k) - \hat{y}_j(k)]\} \\ \hat{y}_i(k) = C_i \hat{x}_i(k) \end{cases} \quad (2)$$

Observe that, in the linear model (1), inputs have been discarded. This is not a limitation because the observer (2) is also linear and the effect of inputs in (1) and (2) cancels out in the dynamics of residuals (defined next) used for fault detection.

We also highlight that the estimator algorithm (2) is distributed, as each local estimator only needs variables from neighboring subsystems in order to update the state estimate. Each local diagnoser computes the *local residual vector*

$$r_i(k) := y_i(k) - \hat{y}_i(k)$$

and uses it to monitor the corresponding subsystem. In our setup, we assume that  $\hat{x}_i(0) = \bar{x}_i$  and, hence, the expectation of the residuals is equal to zero at each iteration, i.e.,  $\mathbb{E}[r_i(k)] = 0$ , at each time  $k$ .

### C. Designing the local thresholds

Given  $\alpha > 1$  and taking advantage of the Chebyshev inequalities, for each  $l$ -th component  $r_{i,l}$  of the residual  $r_i$  we can write

$$\begin{aligned} \Pr(\mathbb{E}[r_{i,l}] - \alpha\sqrt{\text{Var}[r_{i,l}]} \leq r_{i,l} \leq \mathbb{E}[r_{i,l}] + \alpha\sqrt{\text{Var}[r_{i,l}]}) \\ \geq 1 - \frac{1}{\alpha^2}. \end{aligned}$$

Using Chebyshev inequalities could lead to conservative results in terms of time to achieve fault detection and/or missed detection. Less conservative properties can be exploited by requiring further assumptions on the noises distribution. For example, in the Gaussian case the following property could be used:

$$\begin{aligned} \Pr(\mathbb{E}[r_{i,l}] - \alpha\sqrt{\text{Var}[r_{i,l}]} \leq r_{i,l} \leq \mathbb{E}[r_{i,l}] + \alpha\sqrt{\text{Var}[r_{i,l}]}) \\ = F(\alpha) - F(-\alpha), \quad (3) \end{aligned}$$

where

$$F(n) = \int_{-\infty}^n \frac{1}{\sqrt{2\pi\text{Var}[r_{i,l}]}} \exp\left\{-\frac{(r_{i,l} - \mathbb{E}[r_{i,l}])^2}{2\text{Var}[r_{i,l}]}\right\} dr_{i,l}$$

is the cumulative distribution function of the Gaussian stochastic variable  $r_{i,l}$  at a certain time step  $k$ .

We define the time-varying component-wise threshold

$$\bar{r}_{i,l}(k) = \alpha\sqrt{\text{Var}[r_{i,l}(k)]}. \quad (4)$$

Therefore, since  $\mathbb{E}[r_i(k)] = 0$ , at each time  $k$  and for each component  $l$ , in *healthy conditions* it follows that

$$|r_{i,l}(k)| \leq \bar{r}_{i,l}(k),$$

with probability greater than  $1 - \frac{1}{\alpha^2}$  in the general case with no further assumptions on the noises distribution.

It is now of interest to compute the upper bound  $\bar{r}_i(k)$ , which will be used as a local fault detection threshold for the residual  $r_i(k)$  in order to monitor the  $i$ -th subsystem. The objective is to design a fault detection threshold that can be computed in a *distributed* and *scalable* fashion. As it will be clearer in the following, this is possible at the price of using a suitable upper bound for the variance of the estimation error.

We start our analysis by observing that the local residual can be written as

$$r_i(k) = C_i e_i(k) + v_i(k),$$

where  $e_i(k) = x_i(k) - \hat{x}_i(k)$  is the local estimation error,

whose dynamics is given by

$$e_i(k+1) = \sum_{j \in \mathcal{N}_i} \{(A_{ij} - L_{ij}C_j)e_j(k) - L_{ij}v_j(k)\} + w_i(k). \quad (5)$$

Now, we introduce the following extended vectors

$$\begin{aligned} e &:= \text{col}(e_i, i = 1, \dots, M), \\ v &:= \text{col}(v_i, i = 1, \dots, M), \\ w &:= \text{col}(w_i, i = 1, \dots, M). \end{aligned}$$

Moreover, we define the extended matrices  $A$  and  $L$ , as block-matrices having the  $(i, j)$ -th element equal to  $A_{ij}$  and  $L_{ij}$ , respectively:

$$A := [A_{ij}, i = 1, \dots, M, j = 1, \dots, M].$$

Finally,  $C$  is a block-matrix collecting on the main diagonal the matrices  $C_i$ :

$$C := \text{diag}(C_i, i = 1, \dots, M).$$

Therefore, the dynamics of the extended estimation error can be described as

$$e(k+1) = (A - LC)e(k) - Lv(k) + w(k). \quad (6)$$

The covariance matrix of the extended estimation error

$$\Pi(k+1) := \mathbb{E}[e(k+1)e^\top(k+1)]$$

obeys the recursive equation:

$$\Pi(k+1) = (A - LC)\Pi(k)(A - LC)^\top + LRL^\top + Q, \quad (7)$$

with  $\Pi(0) = \Sigma_i$ . Note that, since the residual for the diagnoser is  $r_i(k) = C_i e_i(k) + v_i(k)$ , its covariance matrix (in healthy mode of behavior) is given by

$$\mathbb{E}[r_i(k)r_i^\top(k)] = C_i \Pi_i(k) C_i^\top + R_i, \quad (8)$$

where  $\Pi_i(k) \in \mathbb{R}^{n_i \times n_i}$  is the  $i$ -th diagonal block of matrix  $\Pi(k)$ . However, equation (7) does not allow for a recursive distributed update, because  $\Pi_i(k+1)$  does not depend in general only on the local and neighbors' dynamics, but may be influenced by all the subsystems in the LSS. To overcome this issue, we introduce the time-varying matrices  $B_i(k)$ ,  $i = 1, \dots, M$ , defined using the following distributed recursive update scheme

$$\begin{aligned} B_i(k+1) = \sum_{j \in \mathcal{N}_i} \left[ (\tilde{A}_{ij} - L_{ij}\tilde{C}_j)B_j(k)(\tilde{A}_{ij} - L_{ij}\tilde{C}_j)^\top \right. \\ \left. + L_{ij}\tilde{R}_jL_{ij}^\top \right] + Q_i, \end{aligned} \quad (9)$$

where, for all  $i, j = 1, \dots, M$ ,  $\tilde{A}_{ij} = \sqrt{\varsigma_j}A_{ij}$ ,  $\tilde{C}_i = \sqrt{\varsigma_i}C_i$ , and  $\tilde{R}_i = \varsigma_i R_i$ , and  $\varsigma_i = \text{card}(\mathcal{S}_i)$  is the cardinality of the set  $\mathcal{S}_i$ .

In next Section, we show that, provided a proper initialization is adopted,  $B_i(k)$  is an upper bound to the local estimation error covariance  $\Pi_i(k)$  for any time  $k$ . This bound can be used

for the computation of the local thresholds as follows:

$$\bar{r}_{i,l}(k) = \alpha \sqrt{[C_i B_i(k) C_i^\top + R_i]_{ll}}, \quad (10)$$

where we denote as  $[M]_{ij}$  the  $(i, j)$ -th element of matrix  $M$ . Moreover, as a by-product, we will see in Section IV that the computation of the local bound for the estimation error variance leads to a scalable design procedure for the estimation gains  $L_{ij}$  allowing for PnP operations.

#### D. The fault detection algorithm

In Algorithm 1, the proposed distributed fault detection method is summarized.

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#### Algorithm 1 Fault detection for subsystem $\Sigma_i$

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Set  $\alpha$ 
Design  $L_{ij}$ ,  $j \in \mathcal{N}_i$  (Algorithm 2)
Acquire  $C_j$ ,  $R_j$ ,  $j \in \tilde{\mathcal{N}}_i$ 
Initialize the estimate  $\hat{x}_i(0)$ 
Compute output estimate  $\hat{y}_i(0) = C_i \hat{x}_i(0)$ 
Initialize  $B_i(0)$ 
Measurements  $y_j(0)$  and estimates  $\hat{x}_j(0)$ ,  $\hat{y}_j(0)$ ,  $j \in \tilde{\mathcal{N}}_i$  are
acquired
Bound matrices  $B_j(0)$ ,  $j \in \tilde{\mathcal{N}}_i$  are acquired
Set  $k = 1$ 
while A fault is not detected do
  Compute the estimates  $\hat{x}_i(k)$  and  $\hat{y}_i(k)$  (Eq. (2))
  Update the bound  $B_i(k)$  (Eq. (9))
  Measurements  $y_i(k)$  are acquired
  Compute  $r_i(k) = y_i(k) - \hat{y}_i(k)$ 
  Compute the threshold  $\bar{r}_i(k)$  (Eq. (10))
  Compare  $r_i(k)$  with  $\bar{r}_i(k)$ 
  if  $r_{i,l}(k) > \bar{r}_{i,l}(k)$  for at least one  $l$  then
    Fault detection
  end if
  if  $\tilde{\mathcal{N}}_i(k) \neq \tilde{\mathcal{N}}_i(k-1)$  then
    Update  $A_{ij}$ ,  $L_{ij}$ ,  $j \in \mathcal{N}_i(k)$  if needed (see Section
    IV-A)
    Acquire new  $C_j$ ,  $R_j$ ,  $j \in \tilde{\mathcal{N}}_i(k)$  if new plug-in
  end if
  Measurements  $y_j(k)$  and estimates  $\hat{x}_j(k)$ ,  $\hat{y}_j(k)$ ,  $j \in \tilde{\mathcal{N}}_i$ 
  are acquired
  Bound matrices  $B_j(k)$ ,  $j \in \mathcal{N}_i$  are acquired
   $k = k + 1$ 
end while

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### III. UPPER BOUND TO THE ERROR COVARIANCE MATRIX AND CONVERGENCE PROPERTIES

We start by showing that  $B_i(k)$  can be used as an upper bound to  $\Pi_i(k)$ , for all  $i = 1, \dots, M$  and for all  $k \geq 1$ . As a consequence, the proposed  $B_i(k)$  is suitable for the definition of a bound for the local fault residual  $r_i$ . We have the following result (the proof is reported in the Appendix).

*Theorem 1:* If we set  $B_i(0) \geq \Sigma_i$ , then, for all  $i = 1, \dots, M$ , it holds that  $B_i(k) \geq \Pi_i(k)$ , for all  $k \geq 0$ .  $\square$

Furthermore, in order to define appropriate fault detection thresholds, we need to analyze the stability properties of the estimation error and of the proposed covariance bound. Next, we give a *centralized* condition guaranteeing that, at the same time, the error dynamics (6) is asymptotically stable and  $B_i(k)$  is bounded for all  $k$ .

Some definitions are now in place. We define, for all  $i, j$ ,  $\tilde{F}_{ij} = (\tilde{A}_{ij} - L_{ij} \tilde{C}_j)$  and the matrix  $\tilde{F}$  as the matrix whose blocks are  $\tilde{F}_{ij}$ . Also, we define the following further matrix.

$$\mathbb{F} = \tilde{F} \odot \tilde{F} = \begin{bmatrix} \tilde{F}_{11} \otimes \tilde{F}_{11} & \dots & \tilde{F}_{1M} \otimes \tilde{F}_{1M} \\ \vdots & \ddots & \vdots \\ \tilde{F}_{M1} \otimes \tilde{F}_{M1} & \dots & \tilde{F}_{MM} \otimes \tilde{F}_{MM} \end{bmatrix} \quad (11)$$

where  $\odot$  denotes the Khatri-Rao product, while  $\otimes$  denotes the Kronecker product [9].

Now we are in a position to state the second main result (see the Appendix for the proof).

*Theorem 2:* If matrix  $\mathbb{F}$  is Schur stable, then

- (i) There exists, for all  $i = 1, \dots, M$ , a matrix  $\tilde{B}_i \geq 0$ , independent of the initial conditions of (9), such that  $B_i(k) \rightarrow \tilde{B}_i$  as  $k \rightarrow +\infty$ ;
- (ii)  $A - LC$  is Schur stable.  $\square$

### IV. SCALABLE DESIGN OF LOCAL ESTIMATORS

The results illustrated in the previous section show that the key condition guaranteeing the effectiveness of the proposed estimation/fault detection scheme is the Schur stability of the matrix  $\mathbb{F}$ . This condition can be checked in a *scalable way* via the next result, which follows from Proposition 2 in [19].

*Proposition 1:* For matrices  $L_{ii}$  such that  $\tilde{F}_{ii}$  is Schur stable, if the following conditions are fulfilled

$$\beta_i = \sum_{j \in \tilde{\mathcal{N}}_i} \sum_{k=0}^{\infty} \|\tilde{F}_{ii}^k \tilde{F}_{ij}\|_{\infty}^2 < 1, \quad \forall i = 1, \dots, M \quad (12)$$

then  $\mathbb{F}$  is Schur stable.  $\square$

The proof can be found in the Appendix.

The scalar  $\beta_i$  in (12) depends only on local information. More specifically, it requires some knowledge about the subsystem  $\Sigma_i$  (matrices  $\tilde{A}_{ii}$ ,  $\tilde{C}_i$  and  $A_{ij}$ ,  $j \in \mathcal{N}_i$ ), the  $i$ -th estimator (matrices  $L_{ii}$  and  $L_{ij}$ ,  $j \in \mathcal{N}_i$ ), and about neighbors  $j \in \mathcal{N}_i$  (matrices  $\tilde{C}_j$  and parameters  $\varsigma_j$ , where the latter are needed for computing matrices  $\tilde{A}_{ij}$ ). In particular, no information about  $\Sigma_j$ ,  $j \neq \mathcal{N}_i$  is required. Therefore,  $\beta_i$  can be computed locally by subsystem  $i$ , after having exchanged information with its neighbors. Similarly, the following design problem can be solved locally and independently of  $\Sigma_j$ ,  $j \notin \mathcal{N}_i$ :

*Problem 1:* Compute matrices  $L_{ij}$ ,  $j \in \mathcal{N}_i$  such that  $\tilde{F}_{ii}$  is Schur stable and (12) holds.

As proposed in [19] for PnP state estimation, instead of computing  $L_{ii}$  and  $L_{ij}$ ,  $j \in \mathcal{N}_i$  simultaneously, one can follow the more conservative (but simplified) procedure described in Algorithm 2. This can be easily implemented using the PnPMPCC toolbox for Matlab (see [18]).

**Algorithm 2** Design of a local estimator for subsystem  $\Sigma_i$  (Problem 1)

**Input:**  $\tilde{C}_i, \mathcal{N}_i, \{\tilde{A}_{ij}\}_{j \in \mathcal{N}_i}$

- (i)  $\forall j \in \tilde{\mathcal{N}}_i$ , compute the matrix  $L_{ij}$  solving the optimization problem

$$\min_{L_{ij}} \|\tilde{F}_{ij}\|_{\infty}. \quad (13)$$

- (ii) Compute  $L_{ii}$  such that  $\tilde{F}_{ii}$  is Schur stable and  $\beta_i < 1$ . If it does not exist **stop**.

This approach is justified by the fact that  $\beta_i$  can be bounded from above as

$$\beta_i = \sum_{j \in \mathcal{N}_i} \sum_{k=0}^{\infty} \|\tilde{F}_{ii}^k \tilde{F}_{ij}\|_{\infty}^2 \leq \sum_{k=0}^{\infty} \|\tilde{F}_{ii}\|_{\infty}^{2k} \sum_{j \in \mathcal{N}_i} \|\tilde{F}_{ij}\|_{\infty}^2. \quad (14)$$

Therefore, matrices  $L_{ij}$  in step (i) of Algorithm 2 minimize the upper bound. It should be noted that (13) can be cast into a linear programming problem, as shown in [4]. According to step (ii) of Algorithm 2, the computation of  $L_{ii}$  can be carried out by solving a nonlinear optimization problem. We refer the reader to [16] for a discussion about some numerical aspects.

#### A. PnP operations

After the addition or the removal of a subsystem, the update of the local state estimators and dynamics (9) might be needed for some subsystems. Next, we detail these changes, showing that they may impact at most on subsystems that are two-hops away from the entering/leaving unit.

In both cases, the starting point is a network of subsystems equipped with observers produced by Algorithm 2. We denote with  $T_{PnP}$  the plug-in/unplugging time and use “+” for quantities that must be used after the plug-in/unplugging event (if it takes place).

For each subsystem  $i$ , we define

$$\rho_i^+ = \frac{\varsigma_i^+}{\varsigma_i},$$

where  $\varsigma_i^+ = \text{card}(\mathcal{S}_i^+)$ . We start noting that once a matrix  $L_{ij}$  has been computed using (13), it never changes. Indeed,  $L_{ij}$  minimizes  $\|\tilde{A}_{ij} - L_{ij}\tilde{C}_j\|_{\infty}$  and, since  $\tilde{A}_{ij}^+ = \sqrt{\rho_j^+} \tilde{A}_{ij}$  and  $\tilde{C}_j^+ = \sqrt{\rho_j^+} \tilde{C}_j$ , it also minimizes  $\|\tilde{A}_{ij}^+ - L_{ij}\tilde{C}_j^+\|_{\infty}$ , irrespectively of  $\rho_j^+$ .

1) *Plug-in:* Let us assume that subsystem  $\Sigma_{M+1}$  needs to be plugged-in and be connected with neighbors  $\tilde{\mathcal{N}}_{M+1}$  and successors  $\tilde{\mathcal{S}}_{M+1}$  (Figure 1 provides an example with  $M+1 = 8$ ,  $\tilde{\mathcal{N}}_8 = \{3\}$  and  $\tilde{\mathcal{S}}_8 = \{1, 2\}$ ).

First, each subsystem  $j \in \mathcal{N}_{M+1}$  sends  $\varsigma_j^+$  to its successors. In order to preserve properties (i) and (ii) of Theorem 2, one must design new estimators through Algorithm 2 for subsystem  $M+1$  and for

- all subsystems in  $\tilde{\mathcal{S}}_{M+1}$ , as they will be affected by new coupling terms (see the dashed blue edges in Figure 1);

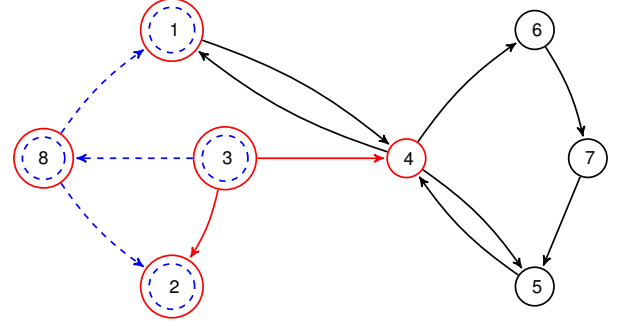


Fig. 1. Plug-in of  $\Sigma_8$  at time  $T_{PnP}$  in a network composed by 7 subsystems. Solid edges: coupling graph of the original network induced by sets  $\mathcal{N}_i$  and  $\mathcal{S}_i$ ,  $i = 1, \dots, 7$ . Dashed blue edges: new coupling links given by  $\tilde{\mathcal{N}}_8 = \{3\}$  and  $\tilde{\mathcal{S}}_8 = \{1, 2\}$ . Dashed blue nodes: subsystems that must successfully run Algorithm 2 (in order to allow the plug-in of  $\Sigma_8$ ) and use new estimators from time  $T_{PnP}$ . Red nodes: subsystems that must update the dynamics (9) from time  $T_{PnP}$ .

- all subsystems  $j \in \mathcal{N}_{M+1}$  because, without changing the gain  $L_{jj}$ , one would have

$$\tilde{F}_{jj}^+ = \sqrt{\rho_j^+} \tilde{F}_{jj} \quad (15)$$

and, since  $\rho_j^+ > 1$ , neither Schur stability of  $\tilde{F}_{jj}^+$  nor  $\alpha_j^+ < 1$  is guaranteed.

If a single instance of the optimization problem in step (ii) of Algorithm 2 is infeasible, the plug-in of  $\Sigma_{M+1}$  is denied. Otherwise it is allowed and new estimators are activated at time  $T_{PnP}$ . Subsystems that must update dynamics (9) and use it from time  $T_{PnP}$ , are, besides  $\Sigma_{M+1}$ :

- subsystems in  $\tilde{\mathcal{S}}_{M+1}$ , as they must include new coupling terms;
- subsystems  $j \in \tilde{\mathcal{N}}_{M+1}$  as  $\rho_j^+ > 1$  and (9) must use the matrix  $\tilde{F}_{jj}^+$  in (15);
- subsystems in  $\mathcal{U}_{M+1} = \cup_{j \in \tilde{\mathcal{N}}_{M+1}} \tilde{\mathcal{S}}_j$ . Indeed, for each  $j \in \tilde{\mathcal{N}}_{M+1}$ , the quantity  $\rho_j^+ > 1$  has been sent to all subsystems  $\ell \in \tilde{\mathcal{S}}_j$  and matrices  $\tilde{A}_{\ell j}$ ,  $\tilde{C}_j$  and  $\tilde{R}_j$ , used in (9) by subsystems  $\ell \in \tilde{\mathcal{S}}_j$ , must be updated by multiplying them by  $\sqrt{\rho_j^+}$ .

Summarizing the above points, all subsystems in the set

$$\mathcal{N}_{M+1} \cup \tilde{\mathcal{S}}_{M+1} \cup \mathcal{U}_{M+1}$$

must update dynamics (9) (see the red nodes in Figure 1). We highlight that no other subsystem in the network need to change the corresponding local estimators or dynamics (9). This motivates the scalability of the plug-in operation.

2) *Unplugging:* We discuss now the unplugging of a subsystem  $\Sigma_u$  at time  $T_{PnP}$ . First, each subsystem  $j \in \mathcal{N}_u$ , having fewer successors after  $\Sigma_u$  unplugging, sends the updated  $\varsigma_j^+$  to its successors. Then,

- subsystems  $i \in \tilde{\mathcal{S}}_u$  can update the local estimator (2) by using the new set  $\tilde{\mathcal{N}}_i^+$  and without changing gains  $L_{ii}$  and  $L_{ij}$ . Indeed,  $\tilde{\mathcal{N}}_i^+$  will have one element less. As a consequence, the sum in (12) is lower. Moreover, in (12),

matrices  $\tilde{F}_{ij}^+ = \tilde{A}_{ij}^+ - L_{ij}\tilde{C}_j^+$  verify

$$\tilde{F}_{ij}^+ = \sqrt{\rho_j^+} \tilde{F}_{ij} \quad (16)$$

with  $\sqrt{\rho_j^+} \leq 1$ . The above conditions guarantee that  $\beta_i^+ < 1$ . Finally, by using the matrix  $L_{ii}$  previously designed, matrix  $\tilde{F}_{ii}^+$  depends on  $\tilde{F}_{ii}$  according to (15) with  $\rho_i^+ \leq 1$ . This shows that Schur stability of  $\tilde{F}_{ii}^+$  is guaranteed.

- the number of successors for subsystems  $j \in \tilde{\mathcal{N}}_u$  is decreased. Therefore, as shown in the previous point, without changing gains  $L_{jj}$  and  $L_{\ell j}$ ,  $\ell \in \mathcal{S}_j^+$ , the matrices  $\tilde{F}_{jj}^+$  are Schur stable and  $\alpha_\ell^+ < 1$ .

It follows that the unplugging of  $\Sigma_u$  can be always performed without spoiling properties (i) and (ii) of Theorem 2, and, similarly to the plug-in operation, subsystems in  $\tilde{\mathcal{N}}_u \cup \tilde{\mathcal{S}}_u \cup \mathcal{U}_u$  will have to update dynamics (9) from time  $T_{PNP}$ .

## V. SIMULATION RESULTS

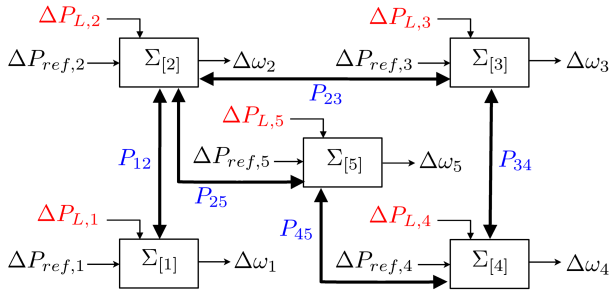


Fig. 2. Power network system of Scenario 2 in [17].

In this section, we provide some simulation results illustrating the effectiveness of the proposed distributed fault detection technique.

As a quite significant case-study, we consider the model of a power network system including a number of power generation areas coupled through tie-lines. In this section we consider scenario 2 in [17], where  $M = 5$  and the neighboring relationships between areas are as the ones illustrated in Figure 2. In this example, neighboring relations are induced by electric lines and they are symmetric since electric power flows in both directions. The dynamics of each power generation area, equipped with primary control and linearized around the equilibrium value for all variables, is described by the following continuous time LTI model [17]:

$$\dot{x}_i(t) = A_{ii}^c x_i(t) + B_i^c u_i + L_i^c \Delta P_{L,i} + \sum_{j \in \mathcal{N}_i} A_{ij}^c x_j, \quad (17)$$

where  $x_i = (\Delta\theta_i, \Delta\omega_i, \Delta P_{m_i}, \Delta P_{v_i})$  is the state<sup>2</sup>,  $u_i = \Delta P_{ref,i}$  is the control input of each area (the deviation of the

<sup>2</sup> $\Delta\theta_i$  is the deviation of the angular displacement of the rotor with respect to the stationary reference axis on the stator,  $\Delta\omega_i$  is the speed deviation of rotating mass from nominal value,  $\Delta P_{m_i}$  represents the deviation of the mechanical power from nominal value, and  $\Delta P_{v_i}$  the deviation of the steam valve position from nominal value.

reference set power from nominal value), and  $\Delta P_{L,i}$  is the local power load. Note that the letter  $\Delta$  is used to denote the deviation from the equilibrium. The matrices of system (17) are

$$A_{ii}^c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{\sum_{j \in \mathcal{N}_i} P_{ij}}{2H_i} & -\frac{D_i}{2H_i} & \frac{1}{2H_i} & 0 \\ 0 & 0 & -\frac{1}{T_{ti}} & \frac{1}{T_{ti}} \\ 0 & -\frac{1}{R_i T_{gi}} & 0 & -\frac{1}{T_{gi}} \end{bmatrix},$$

$$B_i^c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{gi}} \end{bmatrix}, A_{ij}^c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{P_{ij}}{2H_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, L_i^c = \begin{bmatrix} 0 \\ -\frac{1}{2H_i} \\ 0 \\ 0 \end{bmatrix}$$

where the parameters and their numerical values are defined in [17]. They include the definition of the coupling coefficients  $P_i$ , defining the power transferred from area  $j$  to area  $i$  through  $P_{ij}(\Delta\theta_j - \Delta\theta_i)$ , where  $\Delta\theta_j$  and  $\Delta\theta_i$  are the angular displacements of the rotors in area  $j$  and  $i$ .

We consider the local power load profiles reported in Table I.

The AGC control layer design is out of the scope of this paper, so for the sake of simplicity, we set  $\Delta P_{ref,i} = \Delta P_{L,i}$ .

We discretize the process (17) with a sampling interval  $T = 1$  sec leading to the discrete-time model (1) where the matrices  $A_{ii}$ ,  $A_{ij}$  can be easily constructed from (17). The matrix  $C_i$  is

$$C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

For  $i \in \{1, \dots, M\}$ ,  $\mathbb{E}[w_i w_i^T] = Q_i = 10^{-6} I_4$  and  $\mathbb{E}[v_i v_i^T] = R_i = 10^{-6} I_2$  where  $I_k$  is the identity matrix of order  $k$ , representing an approximate Signal to Noise Ratio of 10dB for the output variables.

The state vector is initialized to 0. The gains  $L_{ij}$  of the Luenberger observer proposed in (2), have been computed using Algorithm 2. At the beginning of the simulation only 4 subsystems  $\Sigma_1 - \Sigma_4$  are connected to the network. At time instant  $k = 30$ , a fifth subsystem is plugged-in, connected to  $\Sigma_2$  and  $\Sigma_4$ . The feasibility of the plug-in is checked by means of Algorithm 2 in the neighboring subsystems  $\Sigma_2$  and  $\Sigma_4$  and gains and bound dynamics are updated in the involved subsystems.

At time instant  $k = 45$ , the following fault occurs in area 4: the speed governor time constant  $T_{g4}$  is reduced from 0.1s to 1s, which corresponds to a slower frequency regulation, both in the primary and secondary control layers. After fault detection, subsystem  $\Sigma_4$  is disconnected from the network.

At time instant  $k = 100$ , the following fault occurs in area 5: the inertia constant  $H_5$  is reduced from 10 to 2, which means, from an electrical point of view, that there is a fault in a local generator and hence the faulty area must be isolated for safety reasons, not to propagate faults in the PNS. In order to define the threshold, we set  $\alpha = 2.57$ , which means that we guarantee that the false-alarms rate is lower than 1%, based on Eq. (3). Again, after fault detection, subsystem  $\Sigma_5$  is disconnected from the network.

In Figure 3 we can see residuals and thresholds signals for each measured variable for each area of the PNS. We can

Area	1	1	2	2	3	3	4	4	5	5	5
Time	5	20	15	20	20	30	42	50	50	100	105
$\Delta P_{L_i}$	0.1	-0.22	-0.16	0.12	-0.1	0.1	0.08	-0.22	-0.1	0.2	-0.15

TABLE I  
LOAD PROFILE

see that the local diagnosers are able to detect the fault in Area 4 first, and in Area 5 after, and that the false-alarms are rare events. The fault alarm decision is taken after the residual crosses the corresponding threshold for at least two consecutive time instants and therefore the unplugging of the faulty subsystem is performed. We can see from the figure that, as described in Section IV-A, the bounds and therefore the detection thresholds are updated in the neighboring subsystems after plug-in or unplugging operations.

#### A. Detection delay and false alarms

In this subsection we analyze the detection delay, that is the number of steps required by the proposed algorithm to detect a fault, and the False-Alarms Rate. We consider the same power network system scenario as before, without PnP operations, and with  $M = 5$  connected subsystems. A fault occurs at time  $k_f = 82s$  in area 4: the speed governor time constant  $T_{g4}$  is increased from 0.1s to 2s. We perform 500 experiments characterized by the same noise variance features as before. Figure 4 shows the distribution of the detection delays for the 500 experiments. From the Figure we can see that in most of the cases, detection happens within  $k_f + 1$  and  $k_f + 3$ . Due to the specific considered application example, in few cases (last column on the right), the system reaches a steady state where the considered faults can hardly be detected. We define the False-Alarms Rate (FAR) indicator as follows:

$$FAR = \frac{\text{number of false alarms}}{500 \times M \times p \times T_{\max}} \times 100\%.$$

where  $p = 2$  is the number of residual signals for each diagnoser and  $T_{\max} = 81$  is the duration of the simulation before the occurrence of the fault. From simulations, we obtain an empirical  $FAR = 0.37\%$ , which is lower than the theoretical guaranteed maximum false-alarms probability of 1%, computed from Eq.(3) using  $\alpha = 2.57$ .

#### B. Analysis on the conservativeness of the proposed bound

We now analyze the conservativeness of the proposed local estimation error covariance matrix  $B_i$  defined in (9) on an application example. We consider the same power network system scenario as before, without the occurrence of faults, and with 5 connected subsystems. We compare the proposed bound with the covariance matrix estimate  $P_i$  proposed in [6], and the centralized Kalman Filter error covariance matrix  $\Pi$  defined in (7). In Figure 5 the values of the trace of the considered estimation error covariance matrices are illustrated over time until convergence. It is possible to see that the

proposed bound is conservative with respect to the centralized estimation error covariance<sup>3</sup>. With respect to the covariance matrix  $P_i$  proposed in [6], the proposed bound is comparable in terms of trace. With respect to  $P_i$ , in this paper we have furthermore shown that  $B_i$  represents a bound for the real estimation error covariance at any time step and we have proved its convergence properties.

## VI. CONCLUDING REMARKS

In this paper, we propose a novel distributed fault detection method for interconnected linear systems, allowing the computation of suitable local thresholds guaranteeing that the false-alarms rate is lower than a settable bound. This is achieved by a partition-based distributed estimation method that takes into account the dynamic coupling terms between subsystems. Moreover, a bound on the variance of the estimation error that is computed in a distributed way. Notably, the proposed estimation and fault detection method enjoys scalability features, allowing to remove subsystems and add new ones, provided that suitable plug-in conditions are fulfilled.

As a future work, we are going to analyze the problem of the decomposition of the LSS into subsystems, in order to reduce the conservativeness of the proposed fault detection method and improve the detectability performance. Furthermore, we will investigate the fault isolation problem.

## APPENDIX

The following technical lemma is fundamental for the proofs of the main results of the paper.

**Lemma 1:** Define matrix  $M$  as the matrix whose blocks are  $M_{ij} \in \mathbb{R}^{n_i \times n_j}$  and its sparsity structure is defined as follows:  $M_{ij} = 0$  if  $j \notin \mathcal{N}_i$ . Also, consider the block-diagonal matrix  $\Delta = \text{diag}(\Delta_1, \dots, \Delta_M)$  where, for all  $i = 1, \dots, M$ ,  $\Delta_i \geq 0$  is symmetric. Then, if we define  $\Delta^+ = \text{diag}(\Delta_1^+, \dots, \Delta_M^+)$  where, for all  $i = 1, \dots, M$

$$\Delta_i^+ = \sum_{j \in \mathcal{N}_i} \tilde{M}_{ij} \Delta_j \tilde{M}_{ij}^\top, \quad (18)$$

and  $\tilde{M}_{ij} = \sqrt{\varsigma_j} M_{ij}$ , then  $\Delta^+ \geq M \Delta M^\top$ .

#### Proof of Lemma 1

Define a vector  $v = [v_1^\top, \dots, v_M^\top]^\top$ , where  $v_i \in \mathbb{R}^{n_i}$  for all

<sup>3</sup>It is worth noting that the centralized estimation error covariance, which cannot be computed in a distributed way, is not adequate to compute the fault detection threshold of the designed residual, because it may not represent a bound for the distributed estimation error covariance.



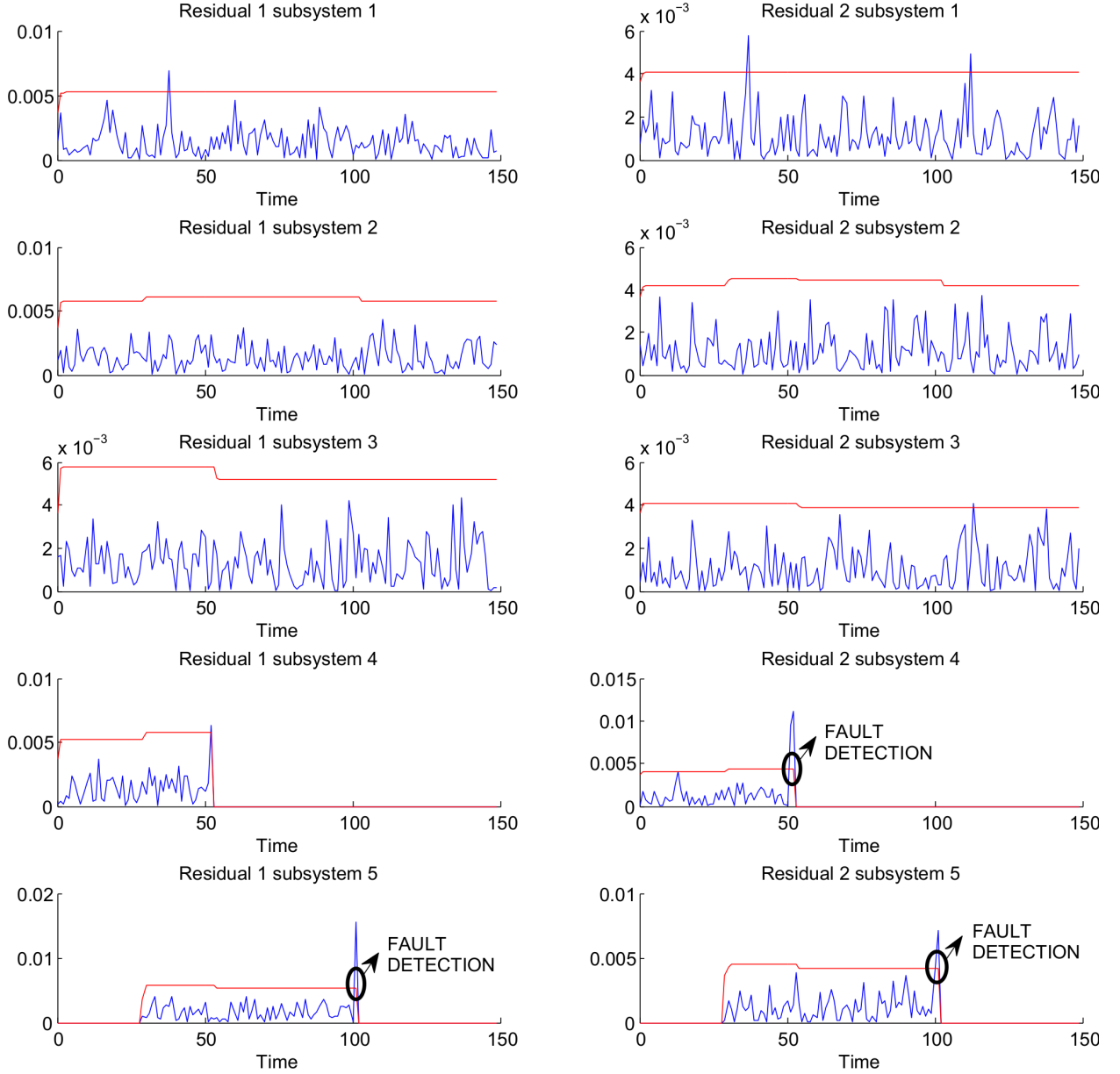


Fig. 3. For each subsystem and each output variable, residual signals (in blue) and thresholds (in red).

$i = 1, \dots, M$ . We have

$$\begin{aligned}
 v^\top M \Delta M^\top v &= \\
 &= \begin{bmatrix} \sum_{i=1}^M v_i^\top M_{i1} & \dots & \sum_{i=1}^M v_i^\top M_{iM} \end{bmatrix} \Delta \begin{bmatrix} \sum_{i=1}^M M_{i1}^\top v_i \\ \vdots \\ \sum_{i=1}^M M_{iM}^\top v_i \end{bmatrix} \\
 &= \sum_{j=1}^M \left( \sum_{i=1}^M w_{ij}^\top \sum_{i=1}^M w_{ij} \right)
 \end{aligned} \tag{19}$$

where  $w_{ij} = \sqrt{\Delta_j} M_{ij}^\top v_i$ . Note that  $w_{ij} = 0$  if  $M_{ij} = 0$ . Moreover, for any fixed  $j$ , the number of nonzero vectors  $w_{ij}$ ,  $i = 1, \dots, M$ , is at most equal to  $\varsigma_j$ . We also have  $\sum_{i=1}^M w_{ij}^\top \sum_{i=1}^M w_{ij} = \sum_{r,s \in \mathcal{S}_j} w_{rj}^\top w_{sj}$ . Note that, since  $\|w_{sj} - w_{rj}\|^2 \geq 0$ ,  $w_{rj}^\top w_{sj} \leq \frac{1}{2}(w_{rj}^\top w_{rj} + w_{sj}^\top w_{sj})$ . Therefore  $\sum_{r,s \in \mathcal{S}_j} w_{rj}^\top w_{sj} \leq \frac{1}{2} \sum_{r,s \in \mathcal{S}_j} (\|w_{rj}\|^2 + \|w_{sj}\|^2) = \varsigma_j \sum_{i \in \mathcal{S}_j} \|w_{ij}\|^2 = \sum_{i \in \mathcal{S}_j} \|v_i\|_{\varsigma_j M_{ij} \Delta_j M_{ij}^\top}^2$ .

From this, it follows that

$$\begin{aligned}
 &\sum_{j=1}^M \left( \sum_{i=1}^M w_{ij}^\top \sum_{i=1}^M w_{ij} \right) \\
 &\leq \sum_{j=1}^M \sum_{i=1}^M \|v_i\|^2 \varsigma_j M_{ij} \Delta_j M_{ij}^\top \\
 &= \sum_{i=1}^M \|v_i\|^2 \sum_{j=1}^M \varsigma_j M_{ij} \Delta_j M_{ij}^\top \\
 &= v^\top \text{diag}(\sum_{j=1}^M \varsigma_j M_{1j} \Delta_j M_{1j}^\top, \dots, \\
 &\quad \dots, \sum_{j=1}^M \varsigma_j M_{Mj} \Delta_j M_{Mj}^\top) v
 \end{aligned}$$

from which (18) readily follows.  $\square$

### Proof of Theorem 1

The proof uses an induction argument. Assume that, for a given  $k \geq 0$ ,  $B_D(k) = \text{diag}(B_1(k), \dots, B_M(k)) \geq \Pi(k)$ . Note that, in view of the definition of  $L_{ij}$  and of the fact that  $C$  is block-diagonal, both  $A - LC$  and  $L$  exhibit the same sparsity structure of  $A$ .

$$(A - LC)\Pi(k)(A - LC)^\top \leq (A - LC)B_D(k)(A - LC)^\top.$$



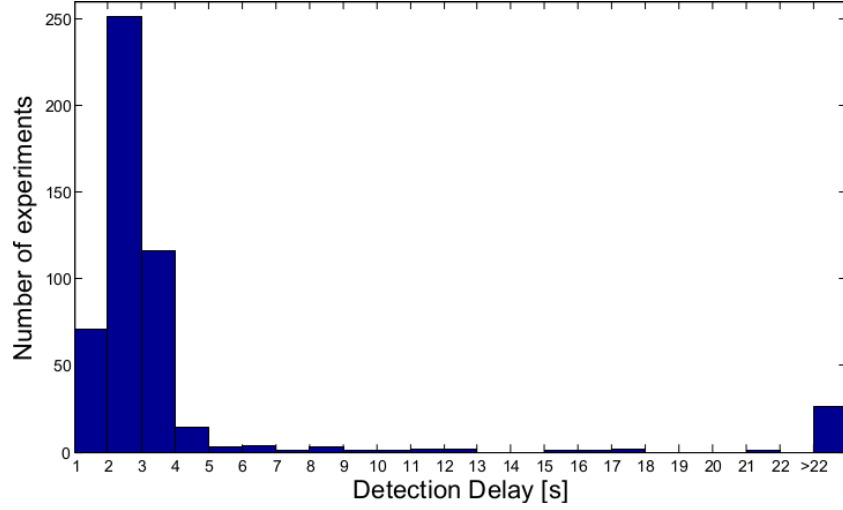


Fig. 4. Distribution of the detection delay for 500 experiments.

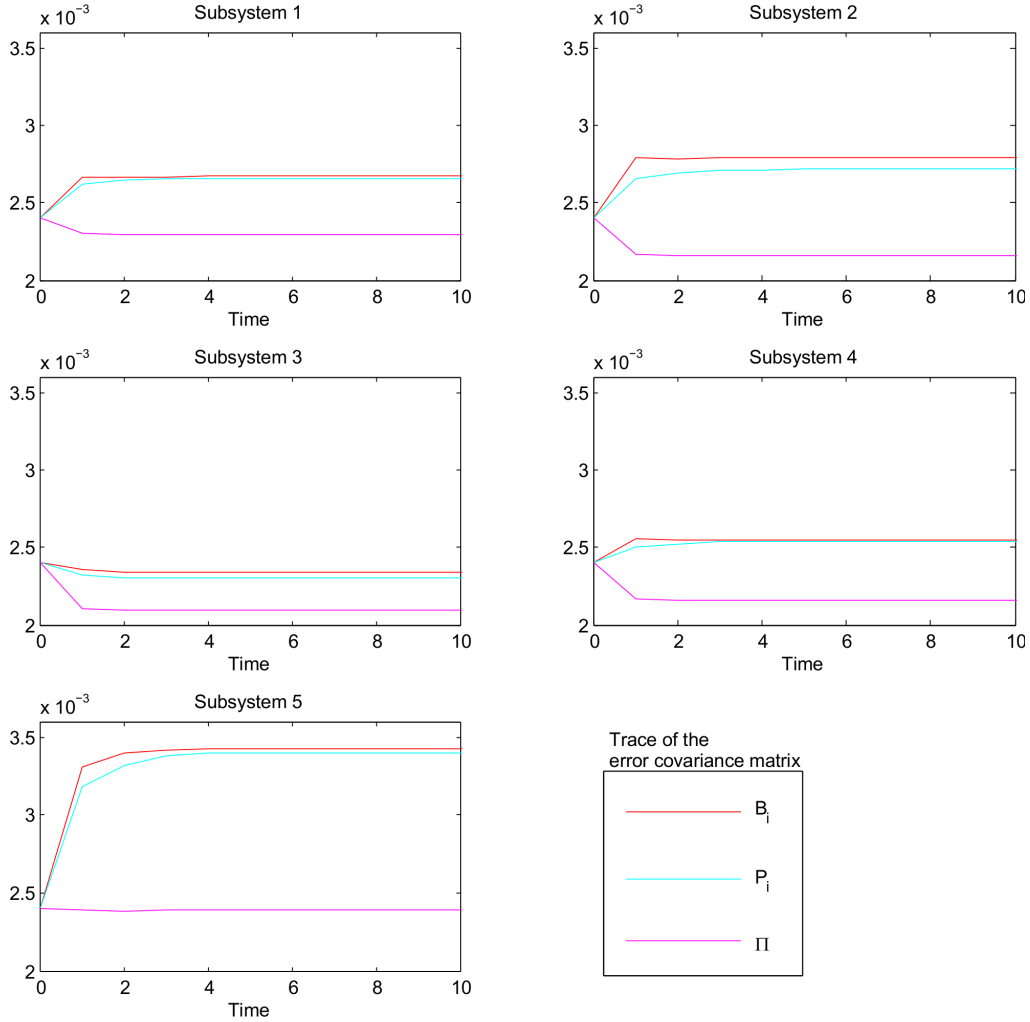


Fig. 5. For each subsystem  $i = 1, \dots, 5$ , trace of the estimation error covariance matrices  $B_i$  defined in (9),  $P_i$  (see [6]), and  $\Pi$  defined in (7).

Since  $B_D(k)$  is block-diagonal we obtain, from Lemma 1, that

$$(A - LC)B_D(k)(A - LC)^\top \leq \text{diag}(B_1^F(k), \dots, B_M^F(k)),$$

where  $B_i^F(k) = \sum_{j \in \mathcal{N}_i} (\tilde{A}_{ij} - L_{ij}\tilde{C}_{ij})B_j(k)(\tilde{A}_{ij} - L_{ij}\tilde{C}_{ij})^\top$ .

Since also  $R$  is block-diagonal we obtain, from Lemma 1, that  $LRL^\top \leq \text{diag}(R_1^L, \dots, R_M^L)$ , where  $R_i^L = \sum_{j \in \mathcal{N}_i} L_{ij}\tilde{R}_jL_{ij}^\top$ . Overall, recalling (7), we obtain that  $\Pi(k+1) = (A - LC)\Pi(k)(A - LC)^\top + LRL^\top + Q \leq \text{diag}(B_1^F(k) +$

$R_1^L, \dots, B_M^F(k) + R_M^L) + Q$ , from which we obtain that  $B_D(k+1) \geq \Pi(k+1)$ . Since  $B_D(0) \geq \Pi(0)$ , by induction we obtain that  $B_D(k) \geq \Pi(k)$  for all  $k > 0$ . Finally, the fact that  $B_i(k) \geq \Pi_i(k)$  for all  $i = 1, \dots, M$  follows from the fact that  $B_i(k) = H_i B_D(k) H_i^\top$  and that  $\Pi_i(k) = H_i \Pi(k) H_i^\top$ , for a suitable matrix  $H_i$ . Therefore  $B_i(k) - \Pi_i(k) = H_i(B_D(k) - \Pi(k))H_i^\top \geq 0$ .  $\square$

### Proof of Theorem 2

*Proof of (i).*

If we define  $\tilde{Q}_i = Q_i + \sum_{j \in \mathcal{N}_i} L_{ij} \tilde{R}_j L_{ij}^\top$  we can write (9) as

$$B_i(k+1) = \sum_{j \in \mathcal{N}_i} \tilde{F}_{ij} B_j(k) \tilde{F}_{ij}^\top + \tilde{Q}_i. \quad (20)$$

Let  $B_i^v(k) = \text{vec}(B_i(k))$  and  $\tilde{Q}_i^v = \text{vec}(\tilde{Q}_i)$ , where  $\text{vec}(X)$  denotes the vectorization of the matrix  $X$  obtained stacking the columns of  $X$  into a single column vector. By recalling the property that, given the matrices  $Y, X, Z$ ,  $\text{vec}(YXZ) = (Y \otimes Z^\top) \text{vec}(X)$  (see [10]), it follows that

$$B_i^v(k+1) = \sum_{j \in \mathcal{N}_i} (\tilde{F}_{ij} \otimes \tilde{F}_{ij}) B_j^v(k) + \tilde{Q}_i^v \quad (21)$$

Overall, we can write (21) in centralized form as

$$B^v(k+1) = \mathbb{F} B^v(k) + \tilde{Q}^v \quad (22)$$

where  $B^v(k)$  and  $\tilde{Q}^v$  are the column vector with blocks  $B_1^v(k), \dots, B_M^v(k)$  and  $\tilde{Q}_1^v, \dots, \tilde{Q}_M^v$ , respectively. Note that (21) is a standard linear discrete-time system. Therefore, if  $\mathbb{F}$  is Schur stable, then  $B^v(k) \rightarrow \bar{B}^v = (I - \mathbb{F})^{-1} \tilde{Q}^v$  as  $k \rightarrow +\infty$ . Moreover, the matrix  $\bar{B}^v$  is independent of the initial conditions of (22). The entries of matrices  $\bar{B}_i$ ,  $i = 1, \dots, M$  are the elements of vector  $\bar{B}^v$ . The fact that  $\bar{B}_i$  are semi positive definite and symmetric for all  $i = 1, \dots, M$ , is guaranteed by construction (9).

*Proof of (ii).* This proof is carried out first in case the pair  $(A, G)$  is stabilizable (where  $GG^\top = Q$ ). From (9) and in view of Lemma 1

$$\bar{B} \geq (A - LC) \bar{B} (A - LC)^\top + LRL^\top + Q \quad (23)$$

where  $\bar{B} = \text{diag}(\bar{B}_1, \dots, \bar{B}_M)$ . Assume, by contradiction, that  $(A - LC)$  is not Schur stable. Therefore, there is at least an eigenvalue/eigenvector pair  $\lambda, v$  of  $(A - LC)^\top$  such that  $(A - LC)^\top v = \lambda v$  and  $|\lambda| \geq 1$ . From (23)

$$v^\top \bar{B} v \geq v^\top (A - LC) \bar{B} (A - LC)^\top v + v^\top Q v + v^\top LRL^\top v$$

from which it follows that  $(1 - |\lambda|^2) v^\top \bar{B} v \geq v^\top Q v + v^\top LRL^\top v$ . Since the right hand side of the latter inequality is  $\geq 0$  and  $|\lambda| \geq 1$ , the only possibility is that  $|\lambda| = 1$ ,  $v^\top Q v = 0$ , and  $L^\top v = 0$ . In view of this,  $A^\top v = \lambda v$  and  $G^\top v = 0$  should hold at the same time which, recalling the PBH test [5], is in contradiction with the assumption that the pair  $(A, G)$  is stabilizable.

In case the pair  $(A, G)$  is not stabilizable, we can apply a similar argument by defining a new matrix sequence  $B_i^*(k)$ ,

$i = 1, \dots, M$ , according to

$$B_i^*(k+1) = \sum_{j \in \mathcal{N}_i} \left\{ (\tilde{A}_{ij} - L_{ij} \tilde{C}_j) B_j^*(k) (\tilde{A}_{ij} - L_{ij} \tilde{C}_j)^\top + L_{ij} \tilde{R}_j L_{ij}^\top \right\} + Q_i^* \quad (24)$$

where  $Q_i^* > 0$  are arbitrary matrices for all  $i = 1, \dots, M$ . Similarly to step (i) of the proof, if  $\mathbb{F}$  is Schur stable then, for all  $i = 1, \dots, M$ ,  $B_i^*(k) \rightarrow \bar{B}_i^* \geq 0$ . Defining  $\bar{B}^* = \text{diag}(\bar{B}_1^*, \dots, \bar{B}_M^*)$ , the former matrix verifies

$$\bar{B}^* \geq (A - LC) \bar{B}^* (A - LC)^\top + LRL^\top + Q^* \quad (25)$$

Since  $Q^* = \text{diag}(Q_1^*, \dots, Q_M^*) > 0$ , its square root  $G^*$  (i.e., defined such that  $G^*(G^*)^\top = Q^*$ ) is full rank  $n$ , and therefore the pair  $(A, G^*)$  is stabilizable. Hence the matrix  $A - LC$  is Schur stable in view of the arguments described right after (23). This concludes the proof of (ii).  $\square$

### Proof of Proposition 1

Using some properties of the Kronecker products (see [9]), one has

$$\begin{aligned} \|\tilde{F}_{ii}^k \tilde{F}_{ij}\|_\infty^2 &= \|(\tilde{F}_{ii}^k \tilde{F}_{ij}) \otimes (\tilde{F}_{ii}^k \tilde{F}_{ij})\|_\infty = \\ &= \|(\tilde{F}_{ii}^k \otimes \tilde{F}_{ii}^k)(\tilde{F}_{ij} \otimes \tilde{F}_{ij})\|_\infty = \\ &= \|(\tilde{F}_{ii} \otimes \tilde{F}_{ii})^k (\tilde{F}_{ij} \otimes \tilde{F}_{ij})\|_\infty = \|\mathbb{F}_{ii}^k \mathbb{F}_{ij}\|_\infty \end{aligned}$$

where  $\mathbb{F}_{ii} = \tilde{F}_{ii} \otimes \tilde{F}_{ii}$  and  $\mathbb{F}_{ij} = \tilde{F}_{ij} \otimes \tilde{F}_{ij}$  denote the blocks composing the matrix  $\mathbb{F}$  in (11). Then, conditions (12) can be written as

$$\sum_{j \in \tilde{\mathcal{N}}_i} \sum_{k=0}^{\infty} \|\mathbb{F}_{ii}^k \mathbb{F}_{ij}\|_\infty < 1, \quad \forall i = 1, \dots, M. \quad (26)$$

Since  $\tilde{F}_{ii}$  is Schur stable,  $\mathbb{F}_{ii}$  has the same property, because each eigenvalue of  $\mathbb{F}_{ii}$  is the product of two eigenvalues of  $\tilde{F}_{ii}$  [9]. Proposition 1 in [19] then shows that conditions (26) imply the Schur stability of  $\mathbb{F}$ .  $\square$

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**Francesca Boem** Francesca Boem received the MSc degree (cum laude) in Management Engineering in 2009 and the PhD degree in Information Engineering in 2013, both from the University of Trieste, Italy. She was Post-Doc at the University of Trieste with the Machine Learning Group from 2013 to 2014. From 2014 to 2018, she was Research Associate at the Department of Electrical and Electronic Engineering, Imperial College London, with the Control and Power Research Group. Since April 2018 Dr. Boem is a Lecturer in the Department of Electronic and Electrical Engineering at University College London (UCL). From 2015 to 2018 she was part of the team at Imperial College which has been awarded the flagship EU H2020-WIDESPREAD-TEAMING project for the development of the EU KIOS Research and Innovation Centre of Excellence, a strategic partnership between University of Cyprus and Imperial College London. Her current research interests include distributed fault diagnosis and fault-tolerant control methods for large-scale networked systems and distributed estimation methods for sensor networks. Dr. Boem is member of the IFAC Technical Committee 6.4 ("Fault Detection, Supervision & Safety of Technical Processes - SAFEPROCESS") and Associate Editor for the IEEE Control System Society Conference Editorial Board and for the EUCA Conference Editorial Board.

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**Ruggero Carli** Ruggero Carli received the Laurea Degree in Computer Engineering and the Ph.D. degree in Information Engineering from the University of Padova, Padova, Italy, in 2004 and 2007, respectively. From 2008 through 2010, he was a Post-doctoral Fellow with the Department of Mechanical Engineering, University of California at Santa Barbara. He is currently an Associate Professor with the Department of Information Engineering, University of Padova. His research interests include control theory and, in particular, control under communication constraints, cooperative control, and distributed estimation.

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**Marcello Farina** Marcello Farina received the Laurea degree in Electronic Engineering in 2003 and the Ph.D. degree in information engineering in 2007, both from the Politecnico di Milano, Milan, Italy. In 2005, he was a visiting student at the Institute for Systems Theory and Automatic Control, Stuttgart, Germany. He is an Associate Professor with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano. His research interests include distributed and decentralized state estimation and control, stochastic model predictive

control, and applications, e.g., mobile robots, sensor networks, and energy supply systems.

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**Thomas Parisini** Thomas Parisini received the Ph.D. degree in Electronic Engineering and Computer Science in 1993 from the University of Genoa. He was with Politecnico di Milano and since 2010 he holds the Chair of Industrial Control and is Director of Research at Imperial College London. He is a Deputy Director of the KIOS Research and Innovation Centre of Excellence, University of Cyprus. Since 2001 he is also Danieli Endowed Chair of Automation Engineering with University of Trieste. In 2009-2012 he was Deputy Rector of University of Trieste. In 2018, he received an Honorary Doctorate from University of Aalborg, Denmark. He authored or co-authored more than 300 research papers in archival journals, book chapters, and international conference proceedings. His more recent research interests include monitoring, diagnosis, control and security of large-scale interconnected nonlinear systems with applications in smart grids, power electronics and industrial process control. He is a co-recipient of the IFAC Best Application Paper Prize of the Journal of Process Control, Elsevier, for the three-year period 2011-2013 and of the 2004 Outstanding Paper Award of the IEEE Trans. on Neural Networks. He is also a recipient of the 2007 IEEE Distinguished Member Award. In 2016 he was awarded as Principal Investigator at Imperial of the H2020 European Union flagship Teaming Project KIOS Research and Innovation Centre of Excellence led by University of Cyprus. In 2012 he was awarded an ABB Research Grant dealing with energy-autonomous sensor networks for self-monitoring industrial environments. Thomas Parisini currently serves as Vice-President for Publications Activities of the IEEE Control Systems Society and during 2009-2016 he was the Editor-in-Chief of the IEEE Trans. on Control Systems Technology and since 2018 he is the Editor in Chief of the European Journal of Control. Since 2017, he is also Editor for Control Applications of Automatica. He is the Chair of the IFAC Technical Committee on Fault Detection, Supervision & Safety of Technical Processes - SAFEPROCESS. He was the Chair of the IEEE Control Systems Society Conference Editorial Board and a Distinguished Lecturer of the IEEE Control Systems Society. He was an elected member of the Board of Governors of the IEEE Control Systems Society and of the European Control Association (EUCA) and a member of the board of evaluators of the 7th Framework ICT Research Program of the European Union. Thomas Parisini is also currently serving as an Associate Editor of the Int. J. of Control and served as Associate Editor of the IEEE Trans. on Automatic Control, of the IEEE Trans. on Neural Networks, of Automatica, and of the Int. J. of Robust and Nonlinear Control. Among other activities, he was the Program Chair of the 2008 IEEE Conference on Decision and Control, the Program Chair of the 2018 European Control Conference, and General Co-Chair of the 2013 IEEE Conference on Decision and Control and of the 2018 IEEE Conference on Control Technology and Applications. Prof. Parisini is a Fellow of the IEEE and of the IFAC.

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**Giancarlo Ferrari-Trecate** Giancarlo Ferrari-Trecate (SM12) received the Ph.D. degree in Electronic and Computer Engineering from the Università degli Studi di Pavia in 1999. Since September 2016 he is Professor at EPFL, Lausanne, Switzerland. In spring 1998, he was a Visiting Researcher at the Neural Computing Research Group, University of Birmingham, UK. In fall 1998, he joined as a Postdoctoral Fellow the Automatic Control Laboratory, ETH, Zurich, Switzerland. He was appointed Oberassistent at ETH, in 2000. In 2002, he joined

INRIA, Rocquencourt, France, as a Research Fellow. From March to October 2005, he worked at the Politecnico di Milano, Italy. From 2005 to August 2016, he was Associate Professor at the Dipartimento di Ingegneria Industriale e dell'Informazione of the Università degli Studi di Pavia. His research interests include decentralised and networked control, plug-and-play control, scalable control of microgrids, modelling and analysis of biochemical networks, hybrid systems and Bayesian learning. Prof. Ferrari-Trecate was a recipient of the 'assegno di ricerca' grant from the University of Pavia in 1999 and the Researcher Mobility Grant from the Italian Ministry of Education, University and Research in 2005. He is currently a member of the IFAC Technical Committee on Control Design and he is on the editorial board of Automatica and Nonlinear Analysis: Hybrid Systems.